

C++ Templates as Partial Evaluation

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Abstract

This paper explores the relationship between C++ templates and partial evaluation. Templates were designed to support generic programming, but unintentionally provided the ability to perform compile-time computations and code generation. These features are completely accidental, and as a result their syntax is awkward. By recasting these features in terms of partial evaluation, a much simpler syntax can be achieved. C++ may be regarded as a two-level language in which types are first-class values. Template instantiation resembles an offline partial evaluator. This paper describes preliminary work toward a single mechanism based on Partial Evaluation which unifies generic programming, compile-time computation and code generation. The language Catat is introduced to illustrate these ideas.

1 Introduction

Templates were added to the C++ language to support generic programming. However, their addition unintentionally introduced powerful mechanisms for compile-time computation and code generation. These mechanisms have proven themselves very useful in generating optimized code for scientific computing applications [13, 16, 22, 23]. Since they are accidental features, their syntax is somewhat awkward. The goal of this paper is to achieve a simpler syntax by recasting these features as partial evaluation. We start by briefly summarizing the capabilities provided by C++ templates, both intended and accidental.

1.1 Generic programming

The original intent of templates was to support generic programming, which can be summarized as “reuse through parameterization”. Generic functions and objects have parameters which customize their behavior. These parameters must be known at compile time (i.e. have static binding). For example, a generic vector class can be declared as:

```

1  template<typename T, int N>
2      class Vector {
3          // some member functions here...
4
5          private:
6              T data[N];
7          };
8
9          // Example use of Vector
10         Vector<int,4> x;

```

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The `Vector` class takes two template parameters (line 1): `T`, a *type parameter*, specifies the element type for the vector; `N`, a *nontype parameter*, is the length of the vector. To use the vector class, template arguments must be provided (line 10). This causes the template to be *instantiated*: an instance of the template is created by replacing all occurrences of `T` and `N` in the definition of `Vector` with `int` and `4`, respectively.

Functions may also be templates. Here is a function template which sums the elements of an array:

```
template<typename T>
T sum(T* array, int numElements)
{
    T result = 0;
    for (int i=0; i < numElements; ++i)
        result += array[i];
    return result;
}
```

This function works for built-in types, such as `int` and `float`, and also for user-defined types provided they have appropriate operators (`=`, `+=`) defined. Templates allow programmers to develop classes and functions which are very customizable, yet retain the efficiency of statically configured code.

1.2 Compile-time computations

Templates can be exploited to perform computations at compile time. This was discovered by Erwin Unruh [19], who wrote a program which produced these errors at compile time:

```
erwin.cpp 10: Cannot convert 'enum' to 'D<2>'
erwin.cpp 10: Cannot convert 'enum' to 'D<3>'
erwin.cpp 10: Cannot convert 'enum' to 'D<5>'
erwin.cpp 10: Cannot convert 'enum' to 'D<7>'
erwin.cpp 10: Cannot convert 'enum' to 'D<11>'
...
...
```

The program tricked the compiler into calculating a list of prime numbers! This capability was quite accidental, but has turned out to be very useful. Here is a simpler example which calculates `pow(X, Y)` at compile time:

```
template<int X, int Y>
struct ctime_pow {
    static const int result = X * ctime_pow<X, Y-1>::result;
};

// Base case to terminate recursion
template<int X>
struct ctime_pow<X, 1> {
    static const int result = X;
};

// Example use:
const int z = ctime_pow<5, 3>::result; // z = 125
```

The first template defines a structure `ctime_pow` which has a single data member `result`. The `static const` qualifiers of `result` make its value available at compile time.

`ctime_pow<X,Y>` refers to `ctime_pow<X,Y-1>`, so the compiler must recursively instantiate the template for $Y, Y-1, Y-2, \dots$ until it hits the base case provided by the second template, which is a *partial specialization*.

Here is an array class which uses `ctime_pow` to calculate the number of array elements needed:

```
template<typename T_numtype, int N_length, int N_dim>
class SquareArray {
    // ...
    static const int numElements = ctime_pow<N_length,N_dim>::result;
    T_numtype data[numElements];
}

// Example use:
SquareArray<float,4,2> x;    // A 4x4 array: will have 16 elements
SquareArray<float,4,3> x;    // A 4x4x4 array: will have 64 elements
```

When the `SquareArray` template is instantiated, `ctime_pow` is used to calculate the array size required. Similar techniques can be used to find greatest common divisors, test for primality, and so on. It is even possible to implement an interpreter for a subset of Lisp which runs at compile time [4].

1.3 Code generation

It turns out that compile-time versions of flow control structures (loops, if/else, case switches) can all be implemented in terms of templates. For example, the definition of `ctime_pow` (Section 1.2) emulates a `for` loop using tail recursion. These compile-time programs can perform code generation by selectively inlining code as they are “interpreted” by the compiler. This technique is called *template metaprogramming* [21]. Here is a template metaprogram which generates a specialized dot product algorithm:

```
template<typename T, int I, int N>
struct meta_dot {
    static inline T f(T* a, T* b)
    { return meta_dot<T,I-1,N>::f(a,b) + a[I]*b[I]; }
};

template<class T, int N>
struct meta_dot<T,0,N> {
    static inline T f(T* a, T* b)
    { return a[0]*b[0]; }
};

// Example use:
float x[3], y[3];
float z = meta_dot<float,2,3>::f(x,y);    // **
```

In the above example, the call to `meta_dot` in line marked `**` results in code equivalent to:

```
float z = a[0]*b[0] + a[1]*b[1] + a[2]*b[2];
```

Head recursion is used to unroll the loop over the vector elements. The syntax for writing such code generators is clumsy. However, the technique has proven very useful in producing specialized algorithms for scientific computing [16, 23].

It is even possible to create and manipulate static data structures at compile time, by encoding them as templates. This is the basis of the *expression templates* technique [20], which creates parse trees of array expressions at compile time. These parse trees are used to generate efficient evaluation routines for array expressions. This technique is the backbone of several libraries for object-oriented numerics [13, 22].

1.4 Traits

The *traits* technique [14] allows programmers to define “functions” which operate on and return *types* rather than data. As a motivating example, consider a generic function which calculates the average value of an array. What should its return type be? If the array contains integers, a floating-point result should be returned. But a floating-point return type obviously will not suffice for a complex-valued array.

The solution is to define a traits class which maps from the type of the array elements to a type suitable for containing their average. Here is a simple implementation:

```
template<typename T>
struct average_traits {
    typedef T T_average;      // default behaviour: T -> T
};

template<>
struct average_traits<int> {
    typedef float T_average; // specialization: int -> float
};
```

An appropriate type for averaging an array of type *T* is given by `average_traits<T>::T_average`. Here is an implementation of `average`:

```
template<class T>
typename average_traits<T>::T_average average(T* array, int N)
{
    typename average_traits<T>::T_average result = sum(array, N);
    return result / N;
}
```

2 Templates as partial evaluation

Partial evaluators [12] regard a program’s data as containing two subsets: static data, which is known at compile time, and dynamic data, which is not known until run time. A partial evaluator evaluates as much of a program as possible (using the static data) and outputs a specialized *residual* program.

To determine which portions of a program may be evaluated, a partial evaluator performs *binding time analysis* to label language constructs and data as static or dynamic. Such a labelled language is called a *two-level language*. For example, a binding-time analysis of some scientific computing code might produce this two-level code fragment:

```
float volumeOfCube(float length)
{
    return pow(length,3);
```

```

float pow(float x, int N)
{
    float y = 1;
    for (int i=0; i < N; ++i)
        y *= x;
    return y;
}

```

in which static constructs have been underlined. A partial evaluator such as CMix [1] would evaluate the static constructs to produce the residual code:

```

float volumeOfCube(float length)
{
    return pow3(length);
}

float pow3(float x)
{
    float y = 1;
    y *= x;
    y *= x;
    y *= x;
}

```

Such specializations can result in substantial performance improvements for scientific code [2, 10].

2.1 C++ as a two-level language

C++ templates resemble a two-level language. Function templates take both template parameters (which have static binding) and function arguments (which have dynamic binding). For example, the `pow` function of the previous example might be declared in C++ as:

```

template<int N>
float pow(float x);      // Calculate pow(x,N)

```

The static data (`N`) is a template parameter, and the dynamic data (`x`) is a function argument. To incorporate template type parameters into this viewpoint, we need to regard types as first-class values. For example, in a declaration such as

```

template<typename X, int Y>
void func(int i, int j);

```

we regard `X` as a piece of data whose value is a type. Since C++ is statically typed, type variables may only have static binding.

It is useful to think of the type of `X` as being `typename`, which can be regarded as a type whose possible values span all types. This point of view has a certain simplifying power: for example, we can now view `typedefs` as assignments between `typename` variables. For example,

```

typedef float float_type;

```

can be regarded as equivalent to the (fictional syntax)

```

typename float_type = float;

```

2.2 Template instantiation as offline PE

Partial evaluation of languages which contain binding-time information is called *offline* partial evaluation. Template instantiation resembles offline partial evaluation: the compiler takes template code (a two-level language) and evaluates those portions of the template which involve template parameters (statically bound values). For example, consider this template class:

```
template<int X>
struct foo {
    static const int result = foo<(X % 2 == 0) ? (X/2) : (3*X+1)>::result;
};

// Base case: X = 1
template<>
struct foo<1> {
    static const int result = 0;
};
```

When `foo<X>` is instantiated, the compiler must determine if `X % 2 == 0` (i.e. whether `X` is even). If true, it instantiates `foo<X/2>`; otherwise `foo<3*X+1>` is instantiated. In theory, this continues until the compiler hits the base case `X=1`.¹

3 A simpler syntax: Catat

We now present preliminary ideas for a single mechanism based on Partial Evaluation which unifies generic programming, compile-time computation, and code generation. To illustrate the ideas, we introduce a (currently hypothetical) language Catat. Catat is a multi-level language based on C++ in which types are first-class values.

3.1 Binding time specifications

Each scope in a Catat program is associated with a default binding time. By default, the global scope has dynamic binding. To indicate statically bound variables, an `@` symbol is appended to the type:

```
int i = 0;    // Dynamic data
int@ j = 0;  // Static data
```

The type `int@` is equivalent to `const int` in C++.

To preserve consistency between the dynamic and static versions of the language, it is necessary to allow multiple levels of binding (or *stages*). The `@` symbol indicates that a variable is bound in the previous stage.

The `@` symbol may also be applied to control constructs:

```
// Calculate N! (factorial) at compile time
int@ N = 5, Nfact = 1;
for@ (int@ i=1; i < N; ++i)
    Nfact *= i;
```

¹Whether this recursion terminates for all `X` is a well-known open problem. In C++, it is impossible to determine if a chain of template instantiations will ever terminate. For this reason, compilers place limits on the depth of template instantiation chains.

Operators such as `=` and `*` are applied at compile-time if their operands are statically bound. Data may flow from static to dynamic constructs, but not vice-versa. This is called cross-stage persistence by Taha and Sheard [18]. For example:

```
int@ i;
int j;

j = i;    // Okay, i is known at runtime
i = j;    // Not okay, j not known at ctime
```

3.2 Functions

Functions in Catat may take a mixture of static and dynamic arguments. We find it convenient to give functions two separate parameters lists, as in C++. Here is an implementation of the `meta_dot` function described earlier:

```
function dot(int@ N, typename@ T)(T* a, T* b) {
    T result = 0;
    for@ (int@ i=0; i < N; ++i)
        result += a[i]*b[i];
    return result;
}
```

Note how much simpler this definition is than its template metaprogram counterpart (Section 1.3).

The function `dot` may be thought of as a *generating extension* or higher-order function. The concept is easier to express in a functional notation:

```
(define dot
  (lambda (static-parms)
    (PE static-parms
      (lambda (dynamic-parms)
        body))))
```

where `(PE parms expr)` performs partial evaluation of `expr` using static parameters `parms`. The use of argument lists of the form `(static-parms) (dynamic-parms)` hints at this idea, and also avoids the parsing difficulty associated with `<>` brackets in C++.

Catat discards the return type specification of C++ and replaces it with the keyword `function`. The return type may result from compile-time calculations, and so must be inferred from the body of the function. As in C++, we allow static parameters to be inferred from dynamic argument types; for example, in the function `average`, `T` can be inferred from the type of the `array` argument.

Functions may be evaluated at either compile-time or run-time. They are not fixed to any stage. For example, given this definition of `pow`:

```
function pow(int X, int N) {
    int result = 1;
    for (int i = 0; i < N; ++i)
        result *= X;
    return result;
}
```

One can invoke `pow` at both run-time and compile-time:

```
int result1 = pow(2,3); // Evaluated at run-time
int@ result2 = pow@(2,3); // Evaluated at compile-time
```

Functions can replace the use of traits classes in C++. Here is a Catat version of `average_traits` (Section 1.4):

```
// Return a type appropriate for averaging an array of T
function average_type(typename T) {
    switch(T) {
        case int:      return float;
        case char:     return float;
        case long int: return double;
        // etc.

        default:       return T;
    }
}
```

This illustrates the usefulness of regarding types as first-class values. Here is the `average` function, recoded in Catat:

```
function average(typename@ T)(T* array, int N) {
    typename@ T_average = average_type@(T);

    // Sum the array, divide by N
    T_average sum = 0;
    for (int i=0; i < N; ++i)
        sum += array[i];
    return sum / N;
}
```

3.3 Specialization

When calls to function templates are encountered during C++ compilation, the template is instantiated. In Catat a similar process would occur, which may be called *specialization*: a partial evaluator produces a residual function by evaluating the static constructs. This function call:

```
int data[10]; // ..
float result = average(int)(data,10);
```

triggers the partial evaluation of `average`; the resulting specialization (translated into C++) might be

```
float average__int(int* array, int N) {
    float sum = 0;
    for (int i=0; i < N; ++i)
        sum += array[i];
    return sum;
}
```

3.4 Classes

Classes in Catat may take static parameters, and contain both static and dynamic data members. For example:

```

1  class SquareArray(typename@ T_numtype, int@ N_length, int@ N_dim) {
2
3      public:
4          SquareArray@()
5          {
6              // Calculate array size needed
7              if@ ((N_dim < 1) || (N_length < 1))
8                  Cata@_error@("N_dim and N_length must be positive.");
9              else@ {
10                  numElements = pow@(N_length,N_dim);
11              }
12
13          SquareArray()
14          {
15              // Initially set elements to zero
16              for (int i=0; i < numElements; ++i)
17                  data[i] = 0;
18          }
19
20      private:
21          static int@ numElements;
22          T_numtype data[numElements];
23      }

```

In this class, there are two constructors: a compile-time constructor `SquareArray@()` and a runtime constructor `SquareArray()`. The `SquareArray@()` constructor is invoked during compilation when the class is specialized. At this time, it can check the static parameters to ensure they are correct; if not, a compile-time error is issued. With the aid of some reflection, this may be the right way to enforce constraints on template parameters (an idea due to Vandevoorde [5]). The constructor `SquareArray()` is invoked at runtime when instances of the class are created.

As with functions, classes have no specified binding time, but may be instantiated at any stage. For example,

```

SquareArray@(int,3,2) x; // Array instantiated at compile-time
SquareArray(int,3,2) y; // Array instantiated at run-time

```

3.5 Thoughts on compiling Cata@

To implement Cata@ as described, one apparently needs both a Cata@-interpreter and a Cata@-compiler. For example, to compile a function f which has static parameters s and dynamic parameters d , these steps would be needed:

1. Use the interpreter to partially evaluate f using the static parameters s :

$$[\![\text{interp}]\!][f, s] = f_s$$

2. Use the compiler to produce native code for the residual function f_s :

$$[\![\text{compiler}]\!][f_s] = f_s^*$$

where \star indicates native code.² This seems wasteful; there would be lots of duplicated effort to create both an interpreter and compiler.

It may be possible to avoid this problem by using an approach similar to that pioneered by the Cmix partial evaluation system [1]. The basic approach is to use a “closure compiler” which uses run-time code generation (RTCG) to compile a single function. RTCG is a bit of a misnomer, since the code generation is being done at compile-time by the compiler.

For example, to evaluate code such as

```
int@ result2 = pow@(2,3); // Evaluated at compile-time
```

One could use the closure compiler (CC) to compile `pow` to native code, and then execute the function with the arguments (2, 3):

$$\begin{aligned} \llbracket CC \rrbracket \text{pow} &= \text{pow}^* \\ \llbracket \text{pow}^* \rrbracket [2, 3] &= 8 \end{aligned}$$

To evaluate a two-level function such as `f(s)(d)`, it must first be transformed into a single-level function. We suggest the term *flattening* for this transform. Flattening turns two-level code into single-level code, by replacing dynamic code with static code that generates syntax trees for the dynamic code. For example, the two-level function

```
float pow(int@ N)(float x)
{
    float result = 1;
    for@ (int@ i = 0; i < N; ++i)
        result *= x;
    return result;
}
```

would be transformed into something like:

```
ASTree powgen(int N)
{
    ASTree func = make_lambda(...);
    ASTree x = make_varref("x");

    func.body().append(make_vardecl(float, "result", 1));
    ASTree result = make_varref("result");

    for (int i=0; i < N; ++i)
        func.body().append(make_op("*=", result, x));

    func.body().append(make_return(result));
    return fn;
}
```

With the aid of this flattening transformation, two-level functions can be compiled without an interpreter:

²The $\llbracket \cdot \rrbracket$ notation is from partial evaluation: if `pow` is a function, then $\llbracket \text{pow} \rrbracket[\text{input}]$ is the result of executing `pow` with `input`. For example, $\llbracket \text{pow} \rrbracket[3, 2] = 9$.

$$\begin{array}{rcl}
 f(s)(d) & \xrightarrow{\text{flatten}} & f_{gen}(s) \\
 \llbracket CC \rrbracket f_{gen} & = & f_{gen}^* \\
 \llbracket f_{gen}^* \rrbracket s & = & f_s \\
 \llbracket CC \rrbracket f_s & = & f_s^*
 \end{array}$$

i.e. first the flattening transform is used to turn $f(s)(d)$ into a single-level generator for f , called f_{gen} . This generator is compiled using CC into native code f_{gen}^* . The native-code version is then executed with the static parameters s , and produces the specialized version f_s . This function is then compiled using CC .

The availability of fast, portable run-time code generation systems such as `vcode` [6] makes this approach to compilation possible.

3.6 Interesting possibilities

Languages with Catat-like abilities raise some interesting possibilities:

3.6.1 Scripting The partial evaluator for Catat needs to contain what is essentially an interpreter to evaluate the static portions of the program. This implies that you get scripting for no extra cost; a Catat program consisting solely of static constructs will be completely interpreted, with no residual code generated. With a little bit of extra work, it ought to be possible to dynamically link to already-compiled Catat programs; this would make it possible to “steer” applications using a natural scripting interface.

3.6.2 Futamura projection Suppose we wrote an interpreter for a domain-specific language (DSL) in Catat. We could design our interpreter to take the input text as a static parameter, and input variables as dynamic parameters. Residualization of the interpreter would result in the DSL code being compiled into the dynamic subset of Catat, via the first Futamura projection [8]. This approach would allow users to incorporate fragments of domain-specific languages into their applications, without sacrificing efficiency.

3.6.3 Reflection and Meta-level Processing A language like Catat may provide a natural environment for implementing reflection and meta-level processing capabilities, since the ability to perform compile-time calculations is there already. Such capabilities would allow programmers to query objects about their methods and members, determine the parameter types of functions, and perhaps even manipulate and generate abstract syntax trees.

4 Related Work

Nielson and Nielson [15] first investigated two-level languages and showed that binding-time analysis can be expressed as a form of type checking. The most closely related work is MetaML, a statically typed multi-level language for hand-writing code generators [18]. MetaML does not appear to address the issue of generic programming. Glück and Jørgensen described a program generator for multi-level specialization [9] which uses a multi-level functional language to represent automatically produced program generators.

Metalevel processing systems address many of the same problems as Catat; they give library writers the ability to directly manipulate abstract syntax trees at compile time. Relevant examples are Xroma [5], MPC++ [11], Open C++ [3], and Magik [7]. These systems are not phrased in terms of partial evaluation or two-level languages; code

generation is generally done by constructing abstract syntax trees. A more closely related system is Catacomb [17], which provides a two-level language for generating runtime library code for parallelizing compilers. However, it does not address issues of generic programming.

The idea of types as first-class values originates in polymorphic or second order typed lambda calculus, and languages based on it.

5 Conclusions

We have shown that C++ with templates may be regarded as a two-level language in which types are first-class values with static binding, and that template instantiation bears a striking resemblance to offline partial evaluation. Languages built on this insight may offer a way to provide generic programming, code generation, and compile-time computation via a single mechanism with simple syntax.

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References

- [1] L. O. Andersen, *Program Analysis and Specialization for the C Programming Language*, PhD thesis, DIKU, University of Copenhagen, May 1994. (DIKU report 94/19).
- [2] A. Berlin and D. Weise, *Compiling scientific code using partial evaluation*, Computer, 23 (1990), pp. 25–37.
- [3] S. Chiba, *A Metaobject Protocol for C++*, in OOPSLA'95, 1995, pp. 285–299.
- [4] K. Czarnecki and U. Eisenecker, *Meta-control structures for template metaprogramming*. <http://home.t-online.de/home/Ulrich.Eisenecker/meta.htm>.
- [5] K. Czarnecki, U. Eisenecker, R. Glück, D. Vandevoorde, and T. L. Veldhuizen, *Generative Programming and Active Libraries*, in Proceedings of the 1998 Dagstuhl-Seminar on Generic Programming, vol. TBA of Lecture Notes in Computer Science, 1998. (in review).
- [6] D. R. Engler, *VCODE : A retargetable, extensible, very fast dynamic code generation system*, in Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation, New York, May 21–24 1996, ACM Press, pp. 160–170.
- [7] ———, *Incorporating application semantics and control into compilation*, in USENIX Conference on Domain-Specific Languages (DSL'97), October 15–17, 1997.
- [8] Y. Futamura, *Partial evaluation of computation process - an approach to a compiler-compiler*, Systems, Computers, Controls, 2 (1971), pp. 45–50.
- [9] R. Glück and J. Jørgensen, *An automatic program generator for multi-level specialization*, Lisp and Symbolic Computation, 10 (1997), pp. 113–158.
- [10] R. Glück, R. Nakashige, and R. Zöchling, *Binding-time analysis applied to mathematical algorithms*, in System Modelling and Optimization, J. Doležal and J. Fidler, eds., Chapman & Hall, 1995, pp. 137–146.
- [11] Y. Ishikawa, A. Hori, M. Sato, M. Matsuda, J. Nolte, H. Tezuka, H. Konaka, M. Maeda, and K. Kubota, *Design and implementation of metalevel architecture in C++ – MPC++ approach*, in Reflection'96, 1996.
- [12] N. D. Jones, *An introduction to partial evaluation*, ACM Computing Surveys, 28 (1996), pp. 480–503.
- [13] S. Karmesin, J. Crottinger, J. Cummings, S. Haney, W. Humphrey, J. Reynders, S. Smith, and T. Williams, *Array design and expression evaluation in POOMA II*, in ISCOPE'98, vol. 1505, Springer-Verlag, 1998. Lecture Notes in Computer Science.
- [14] N. Myers, *A new and useful template technique: “Traits”*, C++ Report, 7 (1995), pp. 32–35.

- [15] F. Nielson and H. R. Nielson, *Two-Level Functional Languages*, Cambridge University Press, Cambridge, Mass., 1992.
- [16] J. G. Siek and A. Lumsdaine, *The Matrix Template Library: A generic programming approach to high performance numerical linear algebra*, in International Symposium on Computing in Object-Oriented Parallel Environments, 1998.
- [17] J. Stichnoth and T. Gross, *Code composition as an implementation language for compilers*, in USENIX Conference on Domain-Specific Languages, 1997.
- [18] W. Taha and T. Sheard, *Multi-stage programming with explicit annotations*, ACM SIGPLAN Notices, 32 (1997), pp. 203–217.
- [19] E. Unruh, *Prime number computation*, 1994. ANSI X3J16-94-0075/ISO WG21-462.
- [20] T. L. Veldhuizen, *Expression templates*, C++ Report, 7 (1995), pp. 26–31. Reprinted in C++ Gems, ed. Stanley Lippman.
- [21] —, *Using C++ template metaprograms*, C++ Report, 7 (1995), pp. 36–43. Reprinted in C++ Gems, ed. Stanley Lippman.
- [22] —, *Arrays in Blitz++*, in ISCOPE'98, vol. 1505 of Lecture Notes in Computer Science, 1998.
- [23] T. L. Veldhuizen and K. Ponnambalam, *Linear algebra with C++ template metaprograms*, Dr. Dobb's Journal of Software Tools, 21 (1996), pp. 38–44.